Flow of Yield-Pseudo-Plastic Fluids through a Concentric Annulus

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Many industrially important non-Newtonian fluids such as drilling fluids and cement slurries exhibit yield-pseudo-plastic behavior. The axial laminar flow of such fluids in concentric annuli has received very little attention to date. This problem was first studied by Hanks (1979), who presented results obtained for the Herschel-Bulkley rheological model. Buchtelova (1988) recently showed that Hanks' numerical results are erroneous in several cases.

The purpose of this work is to present a relatively simple analytical solution to the volumetric flow rate for yield-pseudo-plastic fluids in concentric annuli. Practical examples provided give insight into the use of the present development.

Flow Conditions

This study considers steady, isothermal, incompressible, laminar and axial flow of yield-pseudo-plastic fluids in the annulus between two stationary coaxial cylinders of radii σR and R. The velocity field is assumed to have the form $v_r = v_\theta = 0$ and $v_z = v_z(r)$ with values on the outer and inner walls given as $v_z(R) = v_z(\sigma R) = 0$. For the assumed velocity form, τ_{rz} is the only nonvanishing component of the stress tensor, and hence $\tau = \tau_{rz}$. The shear rate tensor, on the other hand, possesses only one component $\dot\gamma = \dot\gamma_{rz}$ as being equal to the axial velocity gradient ($\dot\gamma_{rz} = dv_z/dr$).

An unsheared plug-flow region exists approximately in the center of the annulus ($\lambda_i R \le r \le \lambda_o R$), as shown in Figure 1. $\lambda_i R$ and $\lambda_o R$ are the two positions at which the yield condition, $\tau_{rz} = \pm\tau_o$, is satisfied.

Rheological Model

The present development is based on a yield-pseudo-plastic rheological model proposed by Robertson and Stiff (1976). For the flow geometry considered here, this model may be expressed as:

$$\begin{aligned} \tau &= K(-dv_z/dr + \gamma_o)^n \qquad && \tau > \tau_o \\ dv_z/dr &= 0 && \tau \le \tau_o \end{aligned} \tag{1}$$

where K, γ_o and n are model parameters that are determined easily from the rheological data. The yield stress of the fluids obeying this model is $\tau_o = K\gamma_o^n$.

Success of this model for describing the yield-pseudo-plastic behavior was proven for drilling fluids and cement slurries (Robertson and Stiff, 1976; Beirute and Fluemerfelt, 1977; Gücüyener, 1983) and for polymer gels (Cloud and Clark, 1983).

Development of Flow Equations

To present the results in a convenient form, the following dimensionless quantities are defined:

$$\begin{gathered} T_{rz} = 2\tau_{rz}/PR;\ T_o = 2\tau_o/PR;\ \Omega = (Q/\pi R^3)(2K/PR)^s \\ \xi = r/R;\ \Gamma_o = \gamma_o(2K/PR)^s;\ \phi = (v_z/R)(2K/PR)^s \end{gathered} \tag{2}$$

The stress distribution is given by the solution of the equations of motion for this geometry (Fredrickson and Bird, 1958) as:

$$T_{rz} = \xi - \frac{\lambda^2}{\xi} \tag{3}$$

Combination of Eqs. 1 and 3 gives the differential equation for ϕ:

$$\left(\pm\frac{d\phi}{d\xi} + T_o^s\right)^n = \pm\left(\frac{\lambda^2}{\xi} - \xi\right) \tag{4}$$

where $(+)$ is used for $\sigma \le \xi \le \lambda_i$ and $(-)$ for $\lambda_o \le \xi \le 1$.

When Eq. 4 is integrated formally using the no slip condition, one obtains the dimensionless velocity distribution as:

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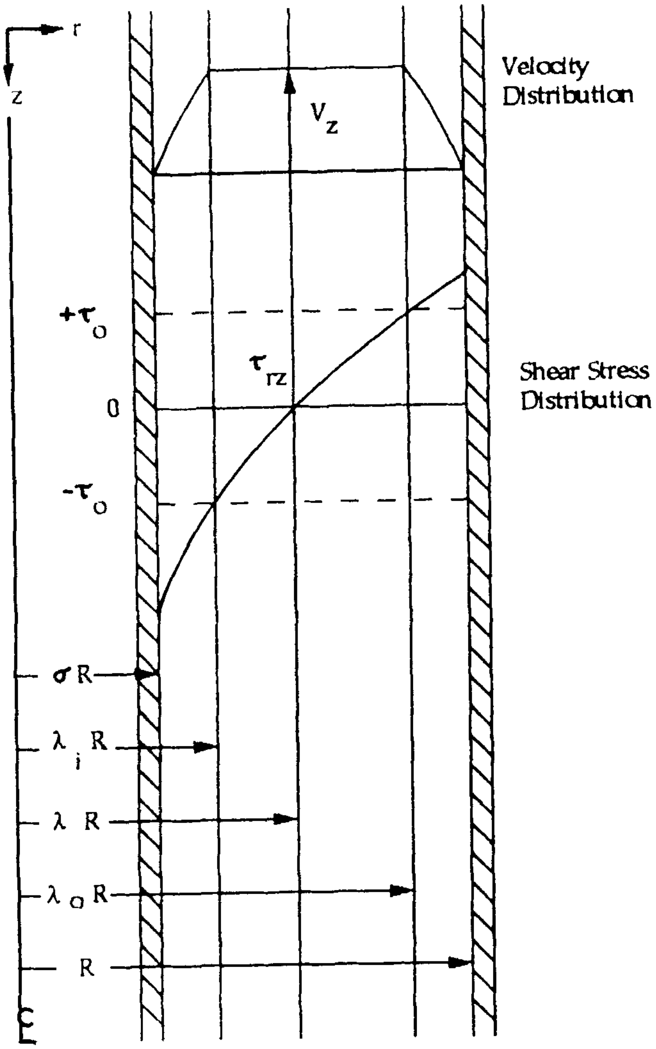

Figure 1. Concentric annulus geometry.

$$\phi_i(\xi)=\int_\sigma^\xi (\lambda^2-\zeta^2)^s\zeta^{-s}d\zeta-(\xi-\sigma)T_o^s \quad (\sigma\le\xi\le\lambda_i) \tag{5}$$

$$\phi_o(\xi)=\int_\xi^1 (\zeta^2-\lambda^2)^s\zeta^{-s}d\zeta-(1-\xi)T_o^s \quad (\lambda_o\le\xi\le 1) \tag{6}$$

λ_i and λ_o are the values of ξ for which the yield condition $T=\pm T_o$ is satisfied. Hence, from Eq. 3,

$$\lambda^2=\lambda_o(\lambda_o-T_o) \tag{7}$$

$$\lambda_i=\lambda_o-T_o \tag{8}$$

The determining equation for λ_o is obtained by setting $\phi_i(\lambda_i)=\phi_o(\lambda_o)$ or

$$\int_\sigma^{\lambda_i}(\lambda^2-\zeta^2)^s\zeta^{-s}d\zeta-\int_{\lambda_o}^1(\zeta^2-\lambda^2)^s d\zeta$$
$$-(2\lambda_o-T_o^s-\sigma-1)T_o^s=0 \tag{9}$$

Equation 9 was solved for λ_o by the application of the Gauss-Legendre quadrature and the Newton-Raphson iteration. Table 1 presents the calculated values of λ_o as a function of σ, n and T_o.

The dimensionless volumetric flow rate, Ω, is given by:

$$\Omega=2\int_\sigma^1\phi(\xi)\xi d\xi \tag{10}$$

Substitution of Eqs. 5 and 6 into Eq. 10, and interchanging the order of integration in the integrals of the resulting equation leads to:

$$\Omega=\int_\sigma^{\lambda_i}(\lambda^2-\xi^2)^{1+s}\xi^{-s}d\xi+\int_{\lambda_o}^1(\xi^2-\lambda^2)^{1+s}\xi^{-s}d\xi$$
$$+\lambda^2(\lambda_i+\lambda_o-\sigma-1)-\frac{1}{3}(\lambda_i^3+\lambda_o^3-\sigma^3-1) \tag{11}$$

An alternative expression for Ω may be obtained by integrating Eq. 10, by parts, as:

$$\Omega=\int_\sigma^{\lambda_i}(d\phi/d\xi)\xi^2 d\xi \tag{12}$$

Since no velocity gradient $(d\phi/d\xi=0)$ exists in the plug-flow region, Eq. 12 may be written in connection with Eq. 4 as:

$$\Omega=\int_\sigma^{\lambda_i}(\lambda^2-\xi^2)^s\xi^{2-s}d\xi+\int_{\lambda_o}^1(\xi^2-\lambda^2)^s\xi^{2-s}d\xi$$
$$+\frac{1}{3}(\lambda_i^3+\lambda_o^3-\sigma^3-1)T_o^2 \tag{13}$$

Integrals in Eq. 13 may be performed by parts, by separating the integrands into ξ^{1+s} and $|\lambda^2-\xi^2|^s\xi d\xi$ to obtain:

$$\Omega=\frac{1}{2}\frac{1}{1+s}\{\lambda_i^{1-s}(\lambda^2-\lambda_i^2)^{1+s}-\sigma^{1-s}(\lambda^2-\sigma^2)^{1-s}$$
$$+(1-\lambda^2)^{1+s}-\lambda_o^{1-s}(\lambda_o^2-\lambda^2)^{1+s}\}+\frac{1}{3}(\lambda_i^3+\lambda_o^3-\sigma^3-1)T_o^s$$
$$-\frac{1}{2}\frac{1-s}{1+s}\left\{\int_\sigma^{\lambda_i}(\lambda^2-\xi^2)^{1+s}\xi^{-s}d\xi+\int_{\lambda_o}^1(\xi^2-\lambda^2)^{1+s}\xi^{-s}d\xi\right\} \tag{14}$$

Combining Eqs. 11 and 14, and rearranging the resulting equation, the following analytical expression is obtained for the dimensionless volumetric flow rate:

$$\Omega=\frac{1}{3+s}\{\lambda_i^{1-s}(\lambda^2-\lambda_i^2)^{1+s}-\sigma^{1-s}(\lambda^2-\sigma^2)^{1+s}+(1-\lambda^2)^{1+s}$$
$$-\lambda_o^{1-s}(\lambda_o^2-\lambda^2)^{1+s}\}-\frac{1-s}{3+s}\lambda^2(\lambda_i+\lambda_o-\sigma-1)T_o^s$$
$$+\frac{1}{3}(\lambda_i^3+\lambda_o^3-\sigma^3-1)T_o^s \tag{15}$$

Table 1. Calculated Values of λ_o (σ, n, T_o) where $\lambda_o = \lambda$ for $T_o = 0$

T_o	n									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					$\sigma = 0.1$					
0.00	0.3442	0.3682	0.3884	0.4052	0.4193	0.4312	0.4412	0.4498	0.4572	0.4637
0.05	0.3701	0.3940	0.4142	0.4309	0.4448	0.4562	0.4656	0.4733	0.4797	0.4851
0.10	0.3978	0.4216	0.4415	0.4579	0.4711	0.4818	0.4903	0.4972	0.5028	0.5075
0.15	0.4273	0.4507	0.4703	0.4860	0.4984	0.5081	0.5158	0.5219	0.5268	0.5308
0.20	0.4584	0.4815	0.5004	0.5152	0.5266	0.5353	0.5421	0.5474	0.5517	0.5552
0.25	0.4912	0.5137	0.5317	0.5454	0.5556	0.5633	0.5692	0.5739	0.5776	0.5806
0.30	0.5254	0.5472	0.5641	0.5765	0.5855	0.5922	0.5973	0.6013	0.6045	0.6070
0.35	0.5611	0.5820	0.5975	0.6085	0.6163	0.6220	0.6263	0.6296	0.6323	0.6345
0.40	0.5980	0.6178	0.6317	0.6412	0.6478	0.6526	0.6562	0.6589	0.6612	0.6629
0.45	0.6361	0.6544	0.6666	0.6746	0.6801	0.6840	0.6869	0.6892	0.6910	0.6924
0.50	0.6752	0.6918	0.7021	0.7087	0.7131	0.7162	0.7186	0.7204	0.7218	0.7229
0.55	0.7152	0.7298	0.7382	0.7434	0.7469	0.7493	0.7511	0.7524	0.7535	0.7544
0.60	0.7559	0.7681	0.7748	0.7787	0.7813	0.7831	0.7844	0.7854	0.7862	0.7868
0.65	0.7970	0.8068	0.8117	0.8145	0.8163	0.8176	0.8185	0.8192	0.8197	0.8202
0.70	0.8384	0.8456	0.8490	0.8508	0.8520	0.8528	0.8534	0.8538	0.8542	0.8544
0.75	0.8798	0.8845	0.8865	0.8875	0.8882	0.8887	0.8890	0.8892	0.8894	0.8896
0.80	0.9209	0.9233	0.9242	0.9247	0.9250	0.9252	0.9253	0.9254	0.9255	0.9256
0.85	0.9611	0.9618	0.9620	0.9622	0.9622	0.9623	0.9623	0.9624	0.9624	0.9624
					$\sigma = 0.2$					
0.00	0.4687	0.4856	0.4991	0.5100	0.5189	0.5262	0.5324	0.5377	0.5422	0.5461
0.05	0.4943	0.5113	0.5247	0.5355	0.5442	0.5512	0.5569	0.5615	0.5653	0.5684
0.10	0.5213	0.5382	0.5515	0.5620	0.5702	0.5766	0.5816	0.5856	0.5889	0.5916
0.15	0.5496	0.5664	0.5794	0.5894	0.5969	0.6026	0.6070	0.6104	0.6132	0.6155
0.20	0.5792	0.5957	0.6083	0.6175	0.6243	0.6292	0.6330	0.6360	0.6383	0.6402
0.25	0.6100	0.6262	0.6381	0.6464	0.6523	0.6565	0.6597	0.6622	0.6642	0.6658
0.30	0.6421	0.6578	0.6686	0.6759	0.6809	0.6845	0.6872	0.6892	0.6909	0.6922
0.35	0.6752	0.6902	0.6998	0.7060	0.7102	0.7132	0.7153	0.7170	0.7183	0.7194
0.40	0.7095	0.7233	0.7316	0.7368	0.7401	0.7425	0.7442	0.7455	0.7466	0.7474
0.45	0.7447	0.7571	0.7640	0.7680	0.7707	0.7725	0.7738	0.7748	0.7756	0.7762
0.50	0.7807	0.7914	0.7967	0.7998	0.8018	0.8031	0.8041	0.8048	0.8054	0.8059
0.55	0.8174	0.8259	0.8299	0.8321	0.8335	0.8344	0.8351	0.8356	0.8360	0.8363
0.60	0.8545	0.8607	0.8634	0.8648	0.8657	0.8663	0.8668	0.8671	0.8674	0.8676
0.65	0.8917	0.8956	0.8972	0.8980	0.8985	0.8989	0.8991	0.8993	0.8995	0.8996
0.70	0.9286	0.9305	0.9313	0.9316	0.9319	0.9320	0.9321	0.9322	0.9323	0.9323
0.75	0.9648	0.9654	0.9655	0.9656	0.9657	0.9657	0.9658	0.9658	0.9658	0.9658
					$\sigma = 0.3$					
0.00	0.5632	0.5749	0.5840	0.5912	0.5970	0.6018	0.6059	0.6093	0.6122	0.6147
0.05	0.5887	0.6004	0.6095	0.6166	0.6223	0.6268	0.6304	0.6333	0.6357	0.6377
0.10	0.6154	0.6271	0.6360	0.6429	0.6481	0.6522	0.6553	0.6578	0.6597	0.6614
0.15	0.6431	0.6547	0.6635	0.6699	0.6745	0.6780	0.6807	0.6827	0.6844	0.6857
0.20	0.6720	0.6834	0.6917	0.6974	0.7015	0.7044	0.7066	0.7084	0.7097	0.7108
0.25	0.7019	0.7130	0.7206	0.7256	0.7290	0.7314	0.7332	0.7346	0.7357	0.7366
0.30	0.7328	0.7434	0.7501	0.7543	0.7570	0.7590	0.7604	0.7615	0.7624	0.7630
0.35	0.7647	0.7745	0.7801	0.7835	0.7856	0.7871	0.7882	0.7891	0.7897	0.7902
0.40	0.7975	0.8061	0.8106	0.8131	0.8147	0.8159	0.8167	0.8173	0.8178	0.8181
0.45	0.8310	0.8381	0.8414	0.8432	0.8444	0.8452	0.8457	0.8461	0.8465	0.8467
0.50	0.8651	0.8703	0.8726	0.8738	0.8745	0.8750	0.8754	0.8757	0.8759	0.8760
0.55	0.8994	0.9028	0.9041	0.9048	0.9052	0.9055	0.9057	0.9058	0.9059	0.9060
0.60	0.9335	0.9352	0.9358	0.9361	0.9363	0.9364	0.9365	0.9366	0.9366	0.9367
0.65	0.9672	0.9676	0.9678	0.9679	0.9679	0.9680	0.9680	0.9680	0.9680	0.9680
					$\sigma = 0.4$					
0.00	0.6431	0.6509	0.6570	0.6617	0.6655	0.6686	0.6713	0.6735	0.6754	0.6770
0.05	0.6686	0.6764	0.6824	0.6871	0.6908	0.6936	0.6959	0.6978	0.6993	0.7005
0.10	0.6950	0.7029	0.7088	0.7132	0.7165	0.7190	0.7209	0.7224	0.7236	0.7246
0.15	0.7224	0.7302	0.7359	0.7399	0.7427	0.7448	0.7463	0.7476	0.7485	0.7493
0.20	0.7508	0.7584	0.7636	0.7671	0.7694	0.7711	0.7723	0.7733	0.7740	0.7746
0.25	0.7801	0.7874	0.7919	0.7947	0.7966	0.7979	0.7989	0.7996	0.8002	0.8006
0.30	0.8103	0.8170	0.8207	0.8229	0.8243	0.8252	0.8259	0.8265	0.8269	0.8272
0.35	0.8413	0.8470	0.8498	0.8514	0.8524	0.8531	0.8536	0.8539	0.8542	0.8545
0.40	0.8730	0.8774	0.8793	0.8804	0.8810	0.8815	0.8818	0.8820	0.8822	0.8823
0.45	0.9051	0.9080	0.9091	0.9097	0.9101	0.9103	0.9105	0.9106	0.9107	0.9108
0.50	0.9372	0.9387	0.9392	0.9395	0.9396	0.9397	0.9398	0.9399	0.9399	0.9399
0.55	0.9689	0.9693	0.9695	0.9695	0.9696	0.9696	0.9696	0.9696	0.9697	0.9697

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Table 1. Calculated Values of λ_a (σ, n, T_o) where λ_o = λ for T_o = 0 (continued)

					$\sigma = 0.5$					
0.00	0.7140	0.7191	0.7229	0.7259	0.7283	0.7303	0.7319	0.7333	0.7345	0.7355
0.05	0.7395	0.7445	0.7483	0.7512	0.7535	0.7553	0.7567	0.7578	0.7587	0.7594
0.10	0.7658	0.7708	0.7745	0.7772	0.7792	0.7806	0.7817	0.7826	0.7833	0.7838
0.15	0.7929	0.7979	0.8013	0.8037	0.8053	0.8064	0.8073	0.8079	0.8084	0.8089
0.20	0.8210	0.8257	0.8287	0.8306	0.8318	0.8327	0.8333	0.8338	0.8341	0.8344
0.25	0.8499	0.8542	0.8565	0.8579	0.8588	0.8594	0.8598	0.8601	0.8604	0.8606
0.30	0.8795	0.8830	0.8847	0.8856	0.8862	0.8866	0.8868	0.8870	0.8872	0.8873
0.35	0.9097	0.9121	0.9131	0.9137	0.9140	0.9142	0.9144	0.9145	0.9146	0.9147
0.40	0.9401	0.9414	0.9419	0.9421	0.9422	0.9423	0.9424	0.9425	0.9425	0.9425
0.45	0.9703	0.9707	0.9708	0.9709	0.9709	0.9709	0.9710	0.9710	0.9710	0.9710
					$\sigma = 0.6$					
0.00	0.7788	0.7818	0.7840	0.7858	0.7872	0.7884	0.7893	0.7902	0.7909	0.7915
0.05	0.8042	0.8072	0.8094	0.8111	0.8124	0.8134	0.8142	0.8148	0.8153	0.8157
0.10	0.8304	0.8333	0.8355	0.8370	0.8380	0.8388	0.8394	0.8398	0.8401	0.8404
0.15	0.8574	0.8602	0.8621	0.8632	0.8640	0.8646	0.8650	0.8653	0.8655	0.8657
0.20	0.8851	0.8877	0.8891	0.8899	0.8904	0.8908	0.8910	0.8912	0.8914	0.8915
0.25	0.9136	0.9156	0.9165	0.9169	0.9172	0.9174	0.9176	0.9177	0.9178	0.9178
0.30	0.9425	0.9436	0.9441	0.9443	0.9444	0.9445	0.9446	0.9446	0.9447	0.9447
0.35	0.9715	0.9718	0.9719	0.9720	0.9720	0.9720	0.9721	0.9721	0.9721	0.9721
					$\sigma = 0.7$					
0.00	0.8389	0.8404	0.8416	0.8426	0.8433	0.8439	0.8444	0.8449	0.8452	0.8455
0.05	0.8642	0.8658	0.8670	0.8679	0.8685	0.8690	0.8693	0.8696	0.8699	0.8700
0.10	0.8904	0.8919	0.8929	0.8936	0.8941	0.8944	0.8946	0.8948	0.8949	0.8950
0.15	0.9172	0.9186	0.9193	0.9197	0.9200	0.9202	0.9203	0.9204	0.9205	0.9205
0.20	0.9447	0.9456	0.9460	0.9462	0.9463	0.9464	0.9464	0.9465	0.9465	0.9465
0.25	0.9725	0.9728	0.9729	0.9729	0.9730	0.9730	0.9730	0.9730	0.9730	0.9730
					$\sigma = 0.8$					
0.00	0.8954	0.8960	0.8965	0.8969	0.8972	0.8975	0.8977	0.8979	0.8980	0.8981
0.05	0.9207	0.9214	0.9218	0.9222	0.9224	0.9226	0.9227	0.9228	0.9228	0.9229
0.10	0.9467	0.9473	0.9476	0.9478	0.9479	0.9480	0.9480	0.9481	0.9481	0.9481
0.15	0.9734	0.9736	0.9737	0.9738	0.9738	0.9738	0.9738	0.9738	0.9738	0.9738
					$\sigma = 0.9$					
0.00	0.9489	0.9491	0.9492	0.9493	0.9493	0.9494	0.9495	0.9495	0.9495	0.9496
0.05	0.9742	0.9744	0.9744	0.9745	0.9745	0.9745	0.9745	0.9745	0.9746	0.9746

Practical Design Applications

This section considers practical applications of the present development. Volumetric flow rate for a given pressure drop can easily be computed from Eq. 15 using Table 1. The pressure drop for a given flow rate, however, cannot be directly calculated due to the implicit character of Eq. 15 with respect to P. The following example illustrates how to handle such problems with the present development.

Example

A drilling fluid, having $\rho = 1{,}020$ kg/m^3, $K = 0.7056$ Pa·s^n, $\gamma_o = 9.76$ s^{-1}, and $n = 0.6$, is to be circulated at a rate of 550 gpm (0.0347 m^3/s) through a 1,500-m-long wellbore annulus between 12.25-in. (0.31115-m)-dia. hole and 5-in. (0.127-m)-OD drill pipe. Find the pressure drop in the annulus.

Solution

The annulus aspect ratio is $\sigma = 5/12.25 = 0.4082$. From Table 1, values of λ_o for various T_o are then obtained by linear interpolation. After subsequent calculation of λ and λ_i from Eqs. 7 and 8, the quantity Ω^n/T_o is obtained via Eq. 15, as given in Table 2. In terms of the given parameters of the system,

$$\frac{\Omega^n}{T_o} = \left(\frac{Q}{\pi R^3 \gamma_o}\right)^n = \left[\frac{8(0.0347)}{\pi(0.31115)^3\,(9.76)}\right]^{0.6} = 0.48612$$

From Table 2, T_o is obtained as 0.2. Using Eq. 2, $P = 4(0.7056)(9.47)^{0.6}/(0.2)(0.3115) = 174.8$ Pa/m. Pressure drop is then (174.8)(1,500) = 262,200 Pa (38 psi).

Conclusion

An analytical flow rate expression is derived for the axial

Table 2. Calculated Values of λ_o and Ω^n/T_o for Selected Values of T_o

T_o	0.05	0.10	0.15	**0.20**	0.25	0.30	0.35	0.40	0.45
λ_o	0.6988	0.7242	0.7500	0.7763	0.8031	0.8304	0.8582	0.8866	0.9154
Ω^n/T_o	2.33075	1.11932	0.70273	**0.48611**	0.35020	0.25517	0.18398	0.12817	0.08318

laminar flow of yield-pseudo-plastic fluids in concentric annuli. The Robertson-Stiff rheological model is used that permits the derivation of such an analytical expression, although this is not possible with the Herschel-Bulkley model due to its mathematical nature.

The position of the plug-flow region and the location of the zero shear surface are strong functions of all the parameters of the model and the annulus aspect ratio. Numerical examples given show that the present theory can easily be utilized in practical applications.

Notation

K = Robertson-Stiff model parameter
L = annulus length
n = Robertson-Stiff model parameter
P = axial pressure gradient
Q = volumetric flow rate
r = radial distance in cylindrical coordinate
R = inner radius of outer cylinder
s = $1/n$
T = dimensionless shear stress
T_o = dimensionless yield stress
v_z = axial velocity
z = axial coordinate

Greek letters

$\dot{\gamma}$ = shear rate tensor
γ_o = Robertson-Stiff model parameter
Γ_o = dimensionless parameter defined in Eq. 2
ζ = dummy variable of integration
θ = angle in cylindrical coordinate
λ = value of ξ for which shear stress is zero
λ_i, λ_o = limits of plug flow region
ξ = dimensionless radial coordinate
ρ = fluid density
σ = ratio of radius of inner cylinder to that of outer cylinder
τ = shear stress tensor
τ_o = yield stress
ϕ = dimensionless velocity defined in Eq. 2
Ω = dimensionless volumetric flow rate defined in Eq. 2

Subscripts

i = range $\sigma \leq \xi \leq \lambda_i$
o = range $\lambda_o \leq \xi \leq 1$
rz = nonzero components of tensor

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